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Sofia Larionova

LIBERAL EDUCATION IN HARMONICS IN PHILO OF ALEXANDRIA*

Writing on ancient "higher" mathematical education is extremely tricky. Most of the time we are left with scanty evidence (if any at all), and more often than not the evidence we have does not allow for practical conclusions. We know quite a lot when it comes to the theory of what it meant to be an educated person in antiquity – the so called educational ideal (or rather, ideals, as there was not just one), – but we are often left to wonder how exactly this theory was put into practice. Mathematical writings, like those of Euclid and Archimedes, are first and foremost scientific treatises, so the practicalities of mathematical education (who studied what, when and to which purpose) are difficult to extract from them. Many other texts that we have are in essence metamathematical: they relate to a certain philosophical or cultural tradition that ascribes certain value to mathematics (the most famous example being Plato's *Republic*). Archeological evidence is scanty and presents many problems, as extant mathematical exercises are quite often difficult to place purpose- and level-wise. More suitable to our goal are encyclopedic texts written for a wider audience to serve as introductions to the μαθήματα (i.e. Theon's *Mathematics Useful for Reading Plato*),[1] and, last but not least, are the testimonies of those with first-hand experience in the area of "higher mathematical education".

But first, the very notion of "higher" education in antiquity needs discussing. The term itself is anachronistic as ancient education was not anything like its modern counterpart. In the context of Greece and Rome, higher education, also known as post-school education, has traditionally been described as the education above the so-called primary and secondary stages, which normally included learning to read and write, playing the lyre, exercising in the palaestra and further instruction in language and

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[1] For a synthesis of the state of research see Bernard 2014, 38–41.

literature.[2] This scholastic curriculum was shown to be too simplistic: in reality, no uniform curriculum existed. Since education was not state-regulated nor standardized, we should not expect to see a uniform trajectory from primary to secondary and then to higher education throughout all the regions of Greco-Roman world and in all time periods (even if we limit ourselves to just one stratum of society).[3] Still, for the lack of a better term, I shall call this stage of education "advanced" or "higher"; the term "liberal education" refers to a non-specialized education in *artes liberales* (or in the ἐγκύκλια, their Greek counterpart), obtained by freemen and meant to produce a well-rounded individual with some exposure to a more or less set number of subjects (usually, those subjects were grammar, dialectic, rhetoric, geometry, arithmetic, astronomy and harmonics).

This advanced stage of education first appeared when the Sophists started their teaching activities.[4] Some of them included mathematical subjects into their curricula, like Hippias of Elis (*Prot*. 318 e) who taught all subjects of quadrivium (i.e. arithmetic, geometry, astronomy and harmonics) – a unity that not long before that had appeared among the Pythagoreans.[5] Plato and Isocrates authored two first theories of education that included mathematical subjects as προπαιδεύματα to the ultimate study goal, i.e. dialectics in Plato (*Resp*. 521 c, 532 b–c) and rhetoric in Isocrates (*Ant*. 261–268).

In this paper, I am going to attempt a description of liberal education in harmonics in the first century AD Alexandria according to the testimony of Philo Judaeus. Philo was not a professional mathematician, but his writing reveals his knowledge – be it not a profound one – of all four subjects of the quadrivium. Non-professional learners, like Philo, deserve our attention as their existence implies public interest in mathematical subjects and certain educational practices. As the bulk of the evidence deals with harmonics, I will focus on education in mathematical theory of music. I will address the following questions: what were the contents of liberal education in harmonics and what were the prerequisites (if there were any).

Despite the fact that μαθήματα made part of the ἐγκύκλιος παιδεία, there is little evidence to them being actually taught at a post-school level, which led some scholars to conclude that they were not studied at all before the time of Augustinus.[6] It is true that, for the most part, Greek

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[2] Marrou 1964, Clarke 1971, Bonner 1977.
[3] Booth 1979, Kaster 1983, Cribiore 2001.
[4] Kühnert 1961.
[5] Huffman 2005, 64.
[6] Hadot 2005.

and Roman education remained predominately literature-oriented. For the first few centuries after the Pythagoreans started teaching mathematics, we have little evidence of actual educational practice. It is in the first century BC that we see wider general public outside of the specialized fields to develop an interest in mathematical subjects. On Roman soil, seemingly out of nowhere appear Varro's *Disciplinae*, the very first encyclopedia and the first treatise to unite trivium and quadrivium under one cover.[7] There is no doubt that Varro was following a Greek tradition and using Greek sources.[8] Philo of Alexandria writes on ἐγκύκλιος παιδεία as someone who had first-hand experience of this education.[9] Private education in mathematical subjects must have been one of few options that were available at the time. Philo's own education would have also been private, considering his family's social standing.[10] Still, Philo is extremely critical of private teachers: some of them cannot explain the subject matter according to their student's capacity (*Post*. 141); others, having encountered a talented student, think themselves better teachers then they really are, and demand higher wages (*Congr*. 127). Philo asserts that a good teacher has to regard the capacity of his student, aim at moderation and bring forward what may improve him (*Post*. 141). But even having a good teacher is not enough, as knowledge is found "only after great labour and with difficulty" (*Som*. 1. 6)[11] and even "having received the doctrines and speculations of wisdom in at his ears from his instructor, <...the student> still is not able to hold it firmly and to embrace it all at once, until he has resolved over in his mind everything which he has heard by the continued exercise of his memory (and this exercise of memory is the cement which connects ideas), and then he impresses the image of it all firmly on his soul" (*Leg.* 4. 107). Still, not every memory is good, only the one exerted on good subjects (*Agr.* 133).

An alternative to live classes was apparently emerging around that time, i.e. there appeared science manuals directed at non-specialists wishing to educate themselves.[12] One might presume that they were free

[7] On Varro's *Disciplinae* and its influence on tradition see Ritschl 1877, Dahlmann 1935, D'Alessandro 1997, Gasti 2017, Simon 1966.

[8] Larionova 2020.

[9] Mendelson 1982.

[10] On Philo's family's position in society see Hadas-Lebel 2012, 27–31.

[11] Translations of Philo are adduced from Yonge 1993. I will further only reference those translations that were in some way modified.

[12] Traces of this tradition are found in Roman literature: following Greek tradition, Varro compiled first Roman encyclopedia that contained all subjects of trivium and quadrivium.

citizens, who were not wealthy enough to pay for private education or to send their children to schools of philosophy or rhetoric. So, when according to Philo was the best time for encyclical studies? In *De congressu erudutionis gratia*, Abraham starts the ἐγκύκλια at about seventeen years old: after childhood,[13] one enters the age of youth (85), but only ten years later one comes to feel the desire for the instruction and is able to benefit from it (88), thus embarking on a long spiritual and educational journey towards "true life". Ideal duration of the studies is said to be seven years, thus stretching from around seventeen to twenty-three (*Ebr.* 52). Still, these numbers have to be taken *cum grano salis,* as they are extremely unlikely to reflect even Philo's own educational journey, not to mention common educational practices of his time (apart from the first stage of education, which did start at about seven years old).[14] In Philo's time, in such a great urban center as Alexandria, education definitely existed in a variety of different shapes and forms. Education that Philo himself obtained and education he describes in his writing would vastly differ from education of those who did not have the means for it.

Ἐγκύκλιος παιδεία is a problematic topic on its own: the term's origin and definition, as well as the history and practices of ἐγκύκλιος παιδεία are all widely disputed.[15] Some of the works tackle various aspects of παιδεία in Philo. F. H. Colson suggested that in Philo ἐγκύκλιος παιδεία "is no longer the preparation for philosophy, but an influence which tempers it and accommodates it to life <...> it creates law and custom as opposed to abstract reason and justice".[16] Walter H. Wagner recognizes different types of παιδεία and analyzes its various aspects, including religious ones.[17] Alan Mendelson believes that according to Philo the ἐγκύκλια had inherent spiritual value and were beneficial on their own, not just as προπαιδεύματα to philosophy.[18] Hent de Vries explores *philosophia ancilla theologiae* motif in Philo's *De congressu*

[13] The length of childhood is not stated anywhere in *Congr.*, but in *Quis her.* 294 this period lasts seven years. On educational stages in Philo see Mendelson 1982.

[14] In Antiquity, educational stages were linked to a theory according to which every seventh year in a person's life was of particular importance. According to Aristotle, the secondary education began at the age of fourteen and ended at the age of twenty-one (*Pol.* 1336 b sqq.). In Plato's *Republic* higher education of the guardians was supposed to start at the age of twenty (*Resp.* 6. 573 b).

[15] Jaeger 1934–1947, Koller 1955, Rechenauer 1994, Fuchs 1962, De Rijk 1965.

[16] Colson 1917, 159.

[17] Wagner 1997, 53–64.

[18] Mendelson 1982, 64–68.

eruditionis gratia.[19] As for harmonics as a part of the ἐγκύκλια, it remains a somewhat under-researched topic. Marrou's classic work contains only one page devoted to the study of harmonics (not just in Philo, mind you, but in general).[20] Certain aspects of music in Philo have been a subject of research. L. H. Feldman's article "Philo's Views on Music" is invaluable, as it brings together musical references scattered all across the Philonic corpus.[21] In an important article, M. Alexandre attempted to evaluate Philo's scientific background, including his knowledge of harmonics.[22] Everett Ferguson concentrates on the practical and philosophical aspects of music.[23]

For a scholar, Philo's time is marked by a deplorable absence of musical sources: after *Elementa harmonica* and *Sectio canonis* at the end of the fourth century BC there is a gap stretching all the way to the new millennium. It is only at the turn of the first to the second century CE that we reencounter musical writings: *Mathematics Useful for Reading Plato* by Theon of Alexandria written in the second century CE[24] and *Enchiridion* by Nicomachus of roughly the same period. Other treatises, probably written sometime in the first century CE, survive in excerpts, e. g. *Pythagorean Elements of Music* by Ptolemais of Cyrene[25] make part of Porphyry's *Commentary on the Harmonics of Ptolemy* (third century CE). Notably, all known treatises from the first and second centuries CE are popular science manuals. Judging by the surviving introductions to these manuals, they were often designed with a specific audience in mind, e. g. those who have not had the opportunity to study mathematics and want to understand the works of Plato (v. intr. to Theon's *Mathematics Useful for Reading Plato*); noble women with no or little previous exposure to the subject (v. intr. to Nicomachus' *Enchiridion*).[26] Ptolemais' treatise, judging by surviving fragments, was written in a form of questions and answers and could have served as a school text, a teacher's aide or a self-study book.

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[19] De Vries 2009, 41.1 – 41.19.

[20] Marrou 1964, 272–273.

[21] Feldman 1996.

[22] Alexandre 1966.

[23] Ferguson 2003.

[24] Theon draws some of his musical material from the astrologer Thrasyllus who lived in the time of Tiberius, and from Adrastus, who was born towards the end of the first century CE.

[25] Her dates are uncertain and might lie between the third century BC and the first century CE. Barker places her "near to the end of this span" (Barker 1989, 230).

[26] For an overview of introductions to various mathematical texts, see Vitrac 2008.

Much like Plato,[27] Philo views μαθήματα, along with the subjects of trivium, as a stepping stone on the way to the ultimate goal, i.e. philosophy (*Congr.* 11), to the proper understanding of which these subjects were meant to contribute (*ib*. 79).[28] People "who are instructed have many more opportunities of prayer than those who are destitute of teachers, and those who are well initiated in encyclical accomplishments have more opportunities than those who are unmusical and illiterate, inasmuch as they from their childhood almost have been imbued with all the lessons of virtue, and temperance, and all kinds of excellence" (*Mut.* 229). Each mathematical subject has potential to turn the reader's soul to virtue: certainty and freedom from deception derive from arithmetic and geometry, as they both deal with proportions and calculations (*Som*. 205), therefore geometry is meant to implant an admiration of justice (*Congr*. 16). In its turn, harmonics "will guide what was previously discordant to concord" (*Congr*. 16)[29] by healing "whatever in us is deficient in rhythm or in moderation, or in harmony, by giving us rhythm, and moderation, and harmony, by means of a polished system of music" (*Cher*. 105).

As beneficial as the study of these subjects was, Philo did not think it wise for students to become professional scientists and devote their

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[27] According to Plato, the study of μαθήματα is "the study that would draw the soul away from the world of becoming to the world of being" (*Resp.* 521 d, tr. Shorey 1935, 147), i.e. it was meant to prepare the soul for the study of dialectics. The future guardians were to devote ten years to various mathematical subjects, including harmonics. Plato compared the scientists who occupied themselves with this science to astronomers, as "their method exactly corresponds to that of the astronomer; for the numbers they seek are those found in these heard concords, but they do not ascend to generalized problems and the consideration which numbers are inherently concordant and which not and why in each case" (531 c, tr. Shorey 1935, 193). In the ideal State, harmonics was to be taught after astronomy, for "like the eyes are framed for astronomy so the ears are framed for the movements of harmony; and these are in some sort kindred sciences, as the Pythagoreans affirm and we admit" (530 d, tr. Shorey 1935, 189). Cf. Archytas: "Indeed, concerning the speed of the stars and their risings and settings as well as concerning geometry and numbers and not least concerning music, they handed down to us a clear set of distinctions. For these sciences seem to be akin" (47 B 1 DK, tr. Huffman 2005, 105–106).

[28] The underlying motive for studying certain mathematical subjects should not be automatically equated with the teaching aims of those providing the education. It is a commonplace in literature that the μαθήματα were studied to facilitate the further understanding of philosophy and the like, but it may not have always been *taught* with this purpose in mind.

[29] Cf. 47 B 3 DK: according to Archytas, the invention of counting put an end to discord (στάσις) and increased concord (ὁμόνοια).

entire lives to them: Philo is disapproving both of people who grow old in geometry, harmonics, etc. (*Congr*. 77),[30] and of those who take up philosophy too early, skipping the encyclical subjects altogether, only to come back to them late und unwillingly and then failing to come back to philosophy (*Ebr*. 51). His believes are in line with the tradition: Isocrates advised young people to explore these subjects for some time and then move on to more useful and important ones (*Antid*. 266–268); Socrates was of opinion that subjects like geometry and astronomy "were enough to occupy a lifetime, to the complete exclusion of many other useful studies" (Xen. *Mem*. 4. 7. 3);[31] an Academic Xenocrates famously said "to someone who had never learnt music, geometry, or astronomy, but nevertheless wished to attend his lectures: 'Go your ways, for you offer philosophy nothing to lay hold of'" (DL 4. 10);[32] an Academic Arcesilaus, who himself was a pupil of the mathematician Autolycus (DL 4. 29), the musician Xanthus, and the geometer Hipponicus (*ib*. 32), is said to be annoyed with people who take up their studies too late (*ib*. 36).

A rough description of an educational journey is contained in *Som*. 1. 205:

> Nevertheless I admire the lover of wisdom for having studied the same art, collecting and thinking fit to weave together many things, though different, and proceeding from different sources, into the same web; for taking the first two elements from the grammatical knowledge imparted to children (ἀπὸ μὲν τῆς παιδικῆς γραμματικῆς), that is to say, reading and writing, and taking from the later education (ἀπὸ δὲ τῆς τελειοτέρας <sc. παιδικῆς>) the skill which is found among poets, and the comprehension of ancient history, and deriving certainty and freedom from deception from arithmetic and geometry, in which sciences there is need of proportions and calculations; and borrowing from music rhyme, and metre, and harmonies, and chromatics, and diatonics, and combined and disjoined melodies; and having derived from rhetoric invention, and language, and arrangement, and memory, and action; and from philosophy, whatever has been omitted in any of these separate branches, and all the other things of which human life consists, he has put together in one most admirably arranged work, combining great learning of one kind with great learning of another kind.[33]

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[30] Cf. Philo, *Post*. 139: "Nothing which is connected with mere professions is akin to virtue".

[31] Tr. Marchant 1923, 349.

[32] Tr. Hicks 1925, 385.

[33] Tr. Yonge 1993 with modifications.

Education starts with children learning to read and write; after that comes analysis of poetic and historic texts, then quadrivium, and finally philosophy. This is not a fully comprehensive curriculum, as some of the subjects are omitted (i.e. astronomy), but all the stages are accounted for. I would suppose that the education described in *Som*. 1. 205 is roughly the education Philo himself received, but also the education that was more or less common in his circle.

Despite the fact that mathematics, in Philo's eyes, borrows definitions from philosophy (*Congr*. 146–147), he still holds it in high esteem, as it was created by the maker of the universe (*Som*. 1. 204), therefore he respects those who study various μαθήματα. Of mathematical subjects, harmonics is the one Philo seems to prefer. Musical passages in his writing are markedly Plato-Pythagorean: he perceives harmonics as a branch of mathematics, not as an independent discipline; as Pythagoreans, he is aware that pitch can be expressed in numbers and that intervals are to be described in number ratios; finally, his world view is also, in certain aspects, Plato-Pythagorean: to him, the study of harmonics is not the ultimate goal, it is a means to understand the cosmic harmony.[34] The frequency and ease with which Philo uses musical metaphors and comparisons is quite astounding and I do not mean to bring all of those together here,[35] as the focus of this paper is on education alone.

Musical education is described in a number of passages (*Congr.* 76; *Mos.* 1. 23; *Leg. all.* 3. 121; *Det.* 18; *Som*. 1. 205). Let us first consider an autobiographical account of Philo's own musical education in *Congr.* 76:

> I was desirous also to form a similar connection with a third, and she was full of good rhythm, harmony, and melody, and was called music. And by her I became the parent of diatonic, and chromatic, and enharmonic,[36] and conjoined and separate melodies,[37] bound to the concords of the fourth, the fifth, and the octave.[38]

After having studied grammar and geometry (*ib*. 74–75), Philo starts studying harmonics. The subject order is not to be taken too literally as all the other branches of encyclical education are completely absent from this account, so it was not meant to be exhaustive. Compare this

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[34] On various approaches to harmonics see Raffa 2020.

[35] See Feldman 1996.

[36] Diatonic, chromatic and harmonic are three different forms of a tetrachord.

[37] When the last note of a tetrachord is also the first in the second tetrachord, then the melodies are conjoined. They are separated when there is an interval of one tone between them.

[38] Tr. Yonge 1993 with modifications.

with the education of Moses,[39] who studies harmonics after arithmetic and geometry, which is a more natural order, as the study of harmonics requires some prior mathematical knowledge (mainly, the knowledge of arithmetic). Along with harmonical theory, Moses learned to play musical instruments (*Mos.* 1. 23):

> Accordingly he speedily learnt arithmetic, and geometry, and the whole science of rhythm and harmony and metre, and the whole of music, by means of the use of musical instruments and by lectures.[40]

Apparently, Philo thought that musical education should include both scientific theory and musical practice, for "it is of no use to study music in an unmusical manner" (*Det.* 18).[41] One would be incomplete without the other, as music, arithmology and astronomy are all closely tied together because of the universal harmony,[42] as seen in *Op.* 126:

> And the power of this number (sc. the number seven) does not exist only in the instances already mentioned, but it also pervades the most excellent of the sciences, the knowledge of grammar and music. For the lyre with seven strings, bearing a proportion to the assemblage of the seven planets, perfects its admirable harmonies, being almost the chief of all instruments, which are conversant about music.

An elementary program in harmonic theory is described in *Leg. all.* 3. 121, where grammar and music – "the most excellent of the sciences" – are put side by side for rhetorical purposes:

> For what would be the advantage of my speaking to a boy distinctly and clearly, and telling him, when I show him the letter A, that it is G, or that the letter E is O? Or what would be the good of a μουσικός pointing out to a pupil who comes to him to learn the rudiments of his art that the enharmonic scale was the chromatic; or the chromatic, the diatonic; or that the highest string was the middle one; or that conjoined sounds were separated; or that the highest tone in the tetrachord scale was a supernumerary note?[43]

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[39] Education of Moses is indicative of what education Philo considered to be ideal.

[40] Tr. Yonge 1993 with modifications.

[41] Cf. Theon, who thought that some understanding of music in instruments (ἐν ὀργάνοις) is useful as a preliminary (47. 6–8).

[42] To Philo, the world itself is a "divine instrument" *(Virt.* 74). On arithmology in Philo see Arndt 1967, Berchman 2013, Moehring 1995, Robbins 1931.

[43] Tr. Yonge 1993 with modifications.

We see that Philo is not only well versed in musical terminology himself (which showcases a certain grasp of the subject[44]) – he also clearly expects that his readers will be able to understand the meaning of these musical terms, as it is with their help that he builds a rhetorical argument, the second part of which should be as clear as the first one, in which he refers to the letters of the alphabet. Secondly, in this piece of evidence Philo refers to the real practice of teaching the basics of harmonics; it also follows from the context that private lessons with a μουσικός specializing in harmonics were not, in Philo's eyes, something out of the ordinary, as they could be put next to a reading lesson. Of course, they still were completely different: much more people could relate to being taught the letters, as opposed to the intricacies of different harmonic scales, but Philo was writing for an audience that could understand the analogy.

This passage, along with the previous ones, provides us with some of the major topics that were studied by non-specialists during harmonic classes. Those included: physical causation of sounds,[45] different notes, strings, intervals, concords, ratios,[46] division of the canon, and, on a more philosophical note, Platonic and Pythagorean ideas relating to cosmic harmony. A comparison to popular musical programs surviving in educational texts from a later period, e.g. Theon's *Mathematics Useful for Reading Plato* or Nicomachus' *Enchiridion*, shows that there were not any major adjustments made to these later programs.

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[44] Still, he sometimes makes mistakes (see Creese 2012, 258–269).

[45] Explained by Philo elsewhere: "the breath being sent from the dominant part of us through the artery called the trachea, is formed in the mouth by the tongue, as by a kind of workman, and being borne outward, and mingled with its kindred air, and having struck it thus harmoniously, completes the mixture of the two powers; for that which sounds together by a combination of different noises is at first adapted to a divisible duad, having one sharp and one flat tone" (*Quod deus* 84).

[46] Ratios per se are not mentioned in the paragraphs in question, but Philo makes extensive use of them in his writing, e.g.: "<...> ten, which is the limit of the number of immensity, around which the numbers wheel and turn as around a goal. Moreover, the number four also comprehends the principles of the harmonious concords in music, that in fours, and in fifths, and the octave, and besides this the double octave from which sounds the most perfect system of harmony is produced. For the ratio of the sounds in fourths is epitritus (ἐπίτριτος); and in fifths hemiolius (ἡμιόλιος); and in the octave that ratio is twofold (διπλάσιος); and in the double octave it is increased fourfold (τετραπλάσιος), all which ratios the number four comprehends. At all events the first, or the epitritus, is the ratio of four to three; the second, or the hemiolius, is that of three to two; the twofold ratio is that of two to one, or four to two: and the fourfold ratio is that of four to one" (*Op.* 47–48, tr. Yonge 1993 with modifications).

Theon of Smyrna, *Mathematics Useful for Reading Plato*	Nicomachus, *Enchiridion*	Philo
2. 2 what is sound	238. 19 – 240. 25 the two forms of vocal sound	opposite sounds: *Plant.* 167 how sound is produced: *Quod deus* 83–85 sharp and flat sounds: *ib.* 23 ear mechanism: *Post.* 103
2. 3–5 what is interval and harmony, notes	241. 20 – 242. 20 notes, harmony 261 what is a note, an interval	intervals, notes, harmony: *Mut.* 87; *Quis her.* 15; *Conf.*, 55–56; *Leg. all.* 3. 121–122
2. 6 concords		*Op.* 48; *Congr.* 76
2. 7–8 tone and semitone	253 tone, semitone	tone: *Quod deus* 24–25; *Som.* 1, 28–29; *Op.* 96; *Leg. all.* 1. 5. 14
2. 9–11 genera of melodies: chromatic, diatonic, enharmonic	261–264 the progression and division of notes according to the three genera	*Leg. all.* 3. 121–122; *Post.* 10; *Agr.* 137; *Congr.* 76; *Som.* 1. 205
2. 19–23 the ratio of proportion	249–252 harmonic proportion, ruling proportion	*Mos.* 115; *Op.* 48, 96; *Spec. leg.* 2. 200
2. 37–39 the tetraktys and the decad		*Op. passim*
2. 40–49 properties of the numbers contained in the decad		*Op. passim*

The table shows that (1) Philo was comfortable with all the main concepts and definitions in the field of mathematical harmonics, and knew how to use those creatively for various rhetorical purposes; (2) Philo definitely was taught Pythagorean harmonics and most certainly had some reference books on the subject at his disposal; (3) as is visible from the table, what he refers to most often are the main harmonic concepts and ratios combined with arithmological number properties; this does not suggest that Philo did not possess deeper knowledge of the

subject – none of his books were devoted to harmonics, so there simply was no need for subtleties.[47]

Furthermore, a program like this one definitely required arithmetic and geometry as prerequisites, because viewing musical intervals as ratios implied knowing how to operate with intervals.[48] Students would be expected to have some experience in adding and subtracting intervals, which mathematically corresponds to multiplying and dividing, e.g. to add a fourth (4/3) to a fifth (3/2) one needs to multiply the ratios: $4/3 \times 3/2 = 2/1$ (i.e. an octave); the addition of equal intervals amounted to raising their ratio to the second power: for example, to add two tones (9/8) one needs to solve for $(9/8)^2 = 81/64$ (i.e. a ditone) and so forth. Just as arithmetic, the ancient study of harmonics most likely included some arithmology, which purported to explain, in the words of Plato, "which numbers are concordant and which are not and why it is so" (*Resp*. 531 c).[49] In Philo's writing, harmonics goes hand in hand with arithmology (*Op.* 95–96):

> The number seven consists of one and two and four, numbers which have two most harmonious ratios, the twofold and the fourfold ratio; the former of which affects the diapason harmony, while the fourfold ratio causes that of the double diapason. It also comprehends other divisions, existing in some kind of yoke-like combination. For it is divided first of all into the number one, and the number six; then into the two and the five; and last of all, into the three and the four. And the proportion of these numbers is a most musical one; for the number six bears to the number one a six-fold ratio, and the six-fold ratio causes the greatest possible difference between existing tones; the distance namely, by which the highest tone is separated from the lowest, as we shall show when we pass on from numbers to the discussion of harmony. Again, the ratio of four to two displays the greatest power in harmony, almost equal to that of the octave, as is most evidently shown in the rules of that art. And the ratio of four to three effects the first harmony, epitritus, which is the fourth.[50]

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[47] More advanced topics that are discussed in Theon and/or in Nicomachus, but are absent in Philo are the following: the diesis (*Exp.* 2. 12), the discovery of the numerical laws of consonances (*Exp.* 2. 12–13, *Ench*. 245. 20 – 248), addition and subtraction of consonances (*Exp.* 2. 13), the leimma (*Exp.* 2. 15–17), epogdoic remainder (*Ench*. 251), the superpartial or sesquipartial relationship (*Exp.* 2. 24), the epimer relationship (*Exp.* 2. 25), the multisuperpartial and polyepimer relationships (*Exp.* 2. 26–28), the foundation of relationships (*Exp*. 2. 29), the difference between the interval and the relationship (*Exp.* 2. 30–32), proportions between three numbers (*Exp*. 2. 33–34), the division of the canon (*Exp*. 2. 35–36), the mean (*Exp*. 2. 50–52, *Ench*. 249–252).

[48] Still, Theon of Smyrna places musical theory right after arithmetic.

[49] Tr. Shorey 1935, 193.

[50] Tr. Yonge 1993 with modifications.

Some acquaintance with geometry was also necessary, as notes were often thought of as points on a straight line, which itself represented an interval. The difference between the genera (diatonic, chromatic, enharmonic)[51] was most probably demonstrated to students either directly on a monochord, or by drawing various lines on the abacus.[52]

As shown above, a set curriculum in harmonics already existed and was implemented in the time of Philo. By his time, it had either already entered the contemporary popular science manuals or was in the process of doing so (as later we see this curriculum already implemented into popular manuals by Theon of Smyrna and Nicomachus).[53]

Sofia Larionova
Saint Petersburg State Pediatric Medical University
salamandrina-na-na@mail.ru

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[51] First introduced by Archytas (A 16, Huffman 2005).

[52] This practice could be one of the major reasons why we have so little archeological evidence in mathematical education.

[53] The same curriculum survived to be taught both in the East and West: Guillaumin 2013, Acerbi 2020.

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This paper aims to analyze the passages of Philo of Alexandria on mathematical education in an attempt to reconstruct post-school mathematical education in the field of mathematical harmonics in the city of Alexandria in the first century CE. Present paper provides evidence that at that time some education in harmonics was received not only by professionals and scientists, but also by other free wealthy citizens. It is shown that the curriculum that became a part of later standard introductions to mathematical sciences, such as "Mathematics useful for reading Plato" by Theon of Smyrna and "Introduction to Harmonics" by Nicomachus of Gerasa, was already being practiced during Philo's time. The topics that formed the basis of this course were the following: the physical causes of sounds, notes, strings, intervals, concords, ratios, separation of the canon, as well as Platonic and Pythagorean ideas about cosmic harmony. To master such a program, some knowledge of arithmetic and geometry was required. Thus, it is shown that to the time of Philo the quadrivium had already developed as an educational unity, where the topics logically followed one another, and the study of each subsequent subject of the quadrivium required basic knowledge from the previous one.

В статье рассматривается распространение практики преподавания квадривиума в эпоху империи на материале сочинений Филона Александрийского (нач. I в. н. э.). Проведен анализ пассажей Филона о математическом образовании и предпринята попытка реконструкции послешкольного математического образования в сфере гармоники в Александрии. Было показано, что в то время гармонике обучались не только профессионалы и ученые-математики, но и другие свободные обеспеченные граждане. Во время Филона уже практиковался *curriculum*, отраженный в более поздних стандартных введениях в математические науки, таких как “Математика, полезная для чтения Платона” Теона Смирнского и “Введение в гармонику” Никомаха Герасского. Основу этого учебного курса составляло изучение следующих тем: физические причины возникновения звуков, ноты, струны, интервалы, консонансы, пропорции, разделение канона, а также платонические и пифагорейские идеи о космической гармонии. Продемонстрировано, что для освоения подобной программы требовались некоторые знания арифметики и геометрии. Таким образом, ко времени Филона квадривиум уже оформился как некое образовательное единство, где темы логически следовали друг за другом, а изучение каждого следующего предмета квадривиума требовало базовых знаний из предыдущего.